Additional Practice

1.

Let f be a function defined by $f(x) = \begin{cases} 2x - x^2 & \text{for } x \le 1, \\ x^2 + kx + p & \text{for } x > 1. \end{cases}$

(a) For what values of k and p will f be continuous and differentiable at $x = 1$?

(b) For the values of k and p found in part (a), on what interval or intervals is f increasing?

(c) Using the values of k and p found in part (a), find all points of inflection of the graph of f. Support your conclusion.

2.

Let f be the function defined by $f(x) = \sin^2 x - \sin x$ for $0 \le x \le \frac{3\pi}{2}$.

(a) Find the x-intercepts of the graph of f.

(b) Find the intervals on which f is increasing.

(c) Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

3.

Consider the function f defined by $f(x) = e^x \cos x$ with domain $[0, 2\pi]$.

(a) Find the absolute maximum and minimum values of $f(x)$.

(b) Find the intervals on which f is increasing.

(c) Find the x-coordinate of each point of inflection of the graph of f.

4.

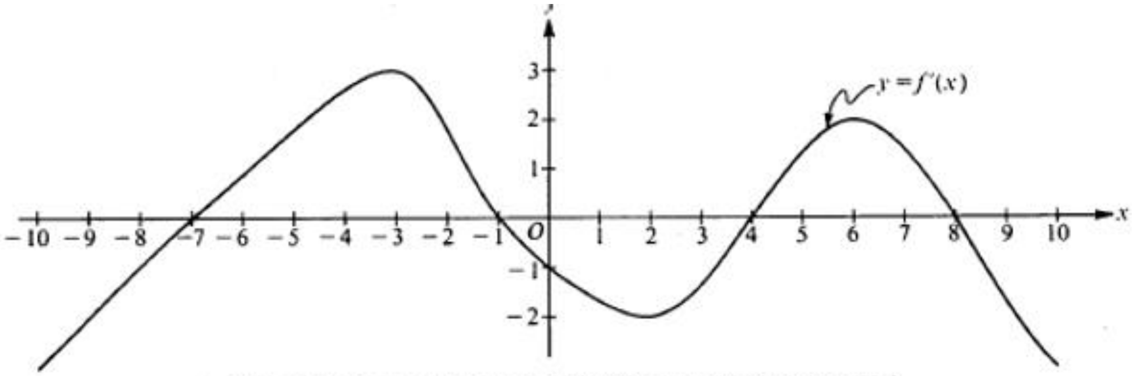
A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \ge 0$ is given by $x(t) = (t-1)^3(2t-3)$.

(a) Find the velocity of the particle at any time $t \ge 0$.

(b) For what values of t is the velocity of the particle less than zero?

(c) Find the value of t when the particle is moving and the acceleration is zero.

5.



Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.

(c) For value of x is the graph of f concave downward?

6.

Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$.

(a) Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.

(b) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.

7.

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{4x - 2xy}{x^2 + y^2 + 1}$.

(b) Write an equation of each horizontal tangent line to the curve.

(c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

8.

A ladder 15 feet long is leaning against a building so that the end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.

9.

A balloon in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261π cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144π cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.

a. At this instant, what is the height of the cylinder?
b. At this instant, how fast is the height of the cylinder increasing?